ASSIGNMENT #4

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. In addition to the true and false section being graded, I will grade one other problem; this will account for 10 points out of 25. The other 15 will be based on completion. **If you would like feedback on a particular problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

(1) Find the transpose of each matrix below:

(a) $$\begin{bmatrix} 1 & 0 & 8 \\ 2 & 6 & 7 \\ 2 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 3 & 9 & 0 & 1 & 7 \\ 7 & 2 & 1 & 5 & 8 \\ 1 & 2 & 1 & 7 & 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(2) Find the inverse of each matrix below:

(a) $$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

(c) $$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(d) $$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

(3) The questions below will guide you on how to solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$ in the case A is an invertible square matrix.

(a) Write down the coefficient matrix for the system of equation

$$\begin{cases} x_1 + x_4 = b_1 \\ 2x_1 - x_2 = b_2 \\ -2x_3 + x_4 = b_3 \\ 2x_2 + 2x_3 = b_4 \end{cases}.$$

(b) Find the inverse of your answer in part (a).

(c) Use your answer in (b) to find a solution to

$$\begin{cases} x_1 + x_4 = 0 \\ 2x_1 - x_2 = 1 \\ -2x_3 + x_4 = 2 \\ 2x_2 + 2x_3 = -1 \end{cases}.$$

(d) Use your answer in (b) to find a solution to

$$\begin{cases} x_1 + x_4 = b_1 \\ 2x_1 - x_2 = b_2 \\ -2x_3 + x_4 = b_3 \\ 2x_2 + 2x_3 = b_4 \end{cases}.$$

where (b_1, b_2, b_3, b_4) is any vector in $\mathbb{R}^4$. Does this agree with your answer in (c)?

(4) Answer the following questions.

(a) Determine if $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is invertible.

(b) Determine if $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible.

(c) Using your answers to parts (a) and (b), do you think is an $n \times n$ matrix consisting of only $1's$ is invertible or not invertible?

(5) List all the statements that are equivalent (i.e the same) as: A is an invertible matrix. **Suggestion: look at the online notes.**

(6) Determine if the following are true or false. No justification necessary.

(a) $(AB)^T = A^T B^T$.

(b) $(AB)^{-1} = B^{-1}A^{-1}$.

(c) $AC = CA$

(d) If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b} \in R^n$.

(e) If AB is invertible, then B is invertible.

(f) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad = bc$, then A is invertible.

(g) $A\mathbf{x} = 0$ has no non-trivial solution, then A is invertible.

(h) If A^T is not invertible, then A is not invertible.